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Abstract

This paper deals with a particular variant of the flowshop scheduling problem motivated by a real case configuration issued from an electro-electronic material industry. The shop floor environment is composed of two parallel semi-lines and a final synchronization operation. The jobs must follow the same technological order through the machines on each parallel semi-line. However, the operations on each semi-line are independent. The final synchronization operation, operated by a dedicated machine, can only start when the job is finished on both semi-lines. The objective is to determine a schedule that minimizes the makespan for a given set of jobs. Since this problem class is NP-hard in the strong sense, constructive heuristic procedures and metaheuristics methods are introduced to achieve optimal or near-optimal solutions. The performances of the proposed GRASP and the Simulated Annealing algorithms are evaluated and compared with the adaptation of two well-known heuristics. Computational experiments show that the proposed metaheuristics provide very good results in low computational times.

Keywords: flowshop scheduling; synchronization operation; heuristic and metaheuristic methods; GRASP algorithm; simulated annealing algorithm

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1. Introduction

In this paper, we consider a variant of the permutation flowshop scheduling problem. The classical permutation flowshop consists of n jobs to be processed among a set of m machines arranged in series. The jobs must follow the same technological order through the machines, and each job has a specific processing time in each machine. The goal is to determine, among all the possible $n!$ sequences, one that optimizes a certain performance measure. The most commonly used is the minimization of the total completion time (makespan).

We analyze a variant of the permutation flowshop problem motivated by a practical application found in the welding sector of an electro-electronics industry. The shop floor environment is composed of two parallel semi-lines and a final synchronization operation. Each semi-line produces one of the halves of a job. These halves are assembled in the final synchronization operation. The halves of the respective jobs must be processed in the same order in each semi-line, which must be followed in the synchronization operation as well. Figure 1 shows a scheme of the studied environment. In this figure, the semi-lines process first the halves of job 2, then those of job 1, and finally those of job 3. This same order is followed in the synchronization operation. An operation in a machine of a semi-line does not need to start at the same time as an operation in the machine of the other semi-line. However, the final synchronization operation of a job can only start when its halves are completed in both semi-lines. The objective is to minimize the makespan.

We consider the cases where the semi-lines have (i) the same number and (ii) different number of machines. A practical example of a process with different number of machines in each semi-line is found in the production of circuit breakers, where one semi-line with two machines processes the contact blade, and the other semi-line with three machines processes the bimetal. When these two semi-products are completed, they are assembled in the synchronization

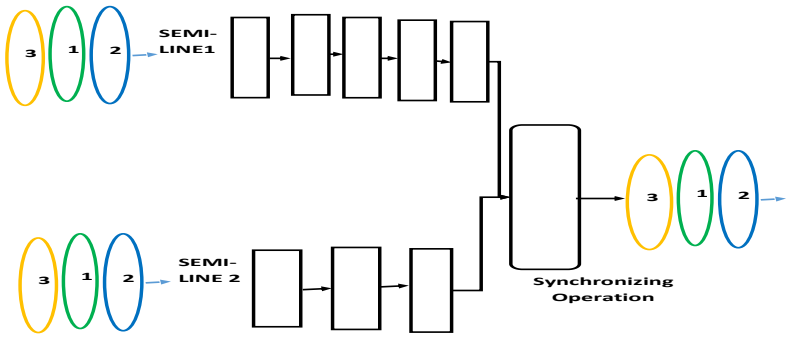


Figure 1: System under study.

operation.

1.1. Illustrative examples

Figure 2 shows the Gantt chart of an optimal sequence for a small illustrative example with 2 jobs, yellow and green, 2 machines in each semi-line l, $l = 1, 2$, and the synchronization operation. We denote by i_k^l, the machine $k = 1, 2$ of semi-line l, and by i_s the synchronization operation. Observe that a half of a job can start in a machine i_k^1 independently of when the other half starts in a machine i_k^2. However, even when a half of a job is completed at semi-line $l = 1$ it must wait for the other half to be completed at semi-line $l = 2$ before the synchronization operation can start.

Let C^* be the optimum makespan of a given instance of the variant of the permutation flowshop under study, and let C^l be the optimum makespan of an instance of the classical permutation flowshop considering the machines of semi-line l along with the synchronization operation. In general we have that $C^* > C^l$, $l = 1, 2$. Let us consider a small example with 3 jobs where each semi-

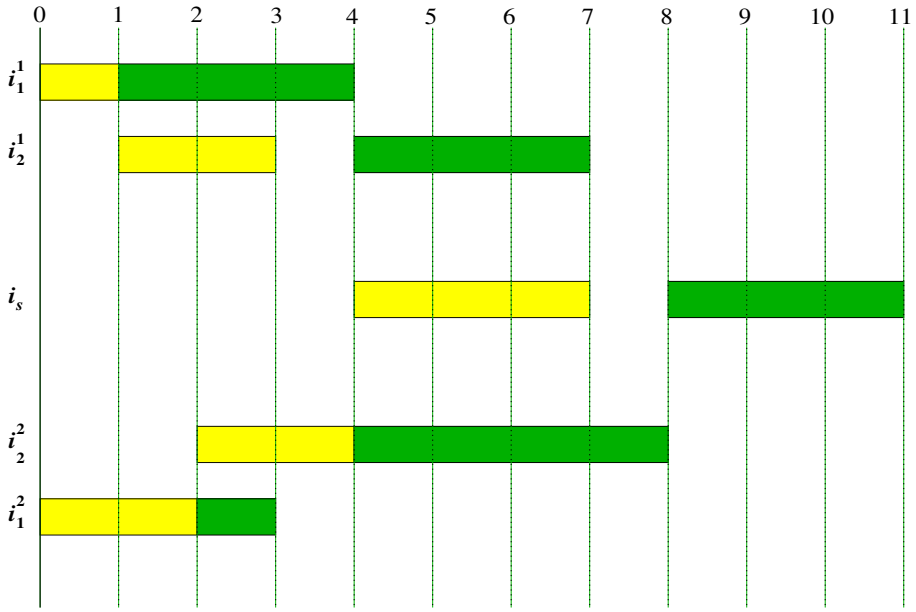


Figure 2: Gantt chart of a small example.

line l has 2 machines, plus the synchronization operation. Table 1 shows the processing times of each job in each machine i_k^l, $k = 1, 2$, and in the synchronization machine i_s. The optimal makespan of the permutation flowshop instance i_1^1, i_2^1, i_s is 94, which can be obtained with the sequence j_1, j_3, j_2. The makespan of this sequence in the problem with two semi-lines and the synchronization operation is 96. Alternatively, the optimal makespan of the permutation flowshop instance i_1^2, i_2^2, i_s is 93, with the sequence j_2, j_1, j_3, and the makespan in the problem with two semi-lines and the synchronization operation is 109. But the optimal makespan of the permutation flowshop with the two semi-lines and the synchronization operation is 95, which can be obtained with the sequence j_3, j_2, j_1 (not optimal for any of the two instances given by the semi-lines considered independently).

This is also observed when the semi-lines have different number of machines. Indeed, let us just consider the instance obtained by removing the first machine i_1^1 from semi-line $l = 1$ in Table 1, i.e., an instance with 1 machine in semi-line

	semi-line $l=1$		semi-line $l=2$		
	i_1^1	i_2^1	i_1^2	i_2^2	i_s
j_1	10	18	14	16	22
j_2	6	37	18	9	25
j_3	5	4	21	8	19

Table 1: An example with the same number of machines in each semi-line.

$l = 1$ and 2 machines in semi-line $l = 2$, plus the synchronization operation. The optimal makespan is still 95, which can be obtained with the same sequence j_3, j_2, j_1.

1.2. Related literature

Many approaches have been proposed for the flowshop problem since Johnson [1] presented the resolution for the flowshop considering two machines. Gupta and Stafford [2] provide a historical perspective of the research in the flowshop problem and its variants. The well-known algorithm of Johnson [1] finds in polynomial time an optimal sequence for a set of n jobs to be processed in $m = 2$ machines. A major difficulty is to find an optimal solution when the number of machines is greater than two, since this problem is known be strongly NP-Hard (Garey et al. [3]). Thus, studies have been developed in the literature of flowshop scheduling considering exact and heuristic techniques as well.

1.2.1. Permutation flowshop

Tseng et al. [4] report a thorough empirical analysis to assess the effectiveness of mixed-integer linear programming (MIP) formulations for the permutation flowshop. We briefly give some more recent examples of the use of MIP models to address flowshop problems in the literature. Frach et al. [5] also present a MIP to solve flowshop problems with a limited number of intermediate buffers. Naderi et al. [6] propose a MIP to minimize the makespan, and the total tardiness in a flowshop environment. Ronconi and Bergin [7] address by MIP the problem

of minimizing the total earliness and tardiness of jobs for the flowshop problem with unlimited and also with zero buffer. Hnaien et al. [8] propose two MIP models for the two-machine flowshop scheduling problem with unavailability constraint in the first machine in order to minimize the makespan. The authors propose a branch and bound algorithm based on new lower bounds and heuristics that performs better than the two MIP models.

Heuristics have been proposed in the literature to obtain good solutions in a short computational time, see, for instance, Mainieri and Ronconi [9], Nawaz et al. [10], Rad et al. [11], and Widmer and Hertz [12]. Johnson's algorithm was adapted by Allahverdi et al. [13] to minimize the total completion time in two-machines flowshop with limited and random processing times. Pan et al. [14] tackle the flowshop problem with zero buffer. In that study, the heuristic of Nawaz et al. [10] exploit specific characteristics of the problem to find good solutions with little computational effort. Allaoui and Artiba [15] aim to minimize the makespan in a two stage hybrid flowshop with a single machine in the first stage and m machines in the second stage. Fernandez-Viagas and Framinan [16] propose efficient tie-breaking mechanisms to be used in the heuristic of Nawaz et al. [10] when dealing with total tardiness.

Metaheuristic approaches have also been proposed in the literature to solve large instances in reasonable computational time. Simulated annealing has been applied by Low et al. [17] and by Nearchou [18] to minimize the makespan in the flowshop problem, and by Mirsanei et al. [19] and by Santosa and Rofiq [20] to the hybrid flowshop problem with m-machines in each stage. GRASP has been applied by Prabhaharan et al. [21]. Shahul Hamid Khan et al. [22] address with GRASP a bicriteria flowshop where the objective is to minimize the weighted sum of makespan and maximum tardiness. Their algorithm was able to outperform a simulated annealing previously proposed by Chakravarthy and Rajendran [23] for the same problem. On the other hand, in the computational experiments conducted by Sivasankaran et al. [24] simulated annealing outperformed GRASP for a single-stage scheduling problem.

1.2.2. Distributed permutation flowshop

In 2010, Naderi and Ruiz [25] introduced a new generalization of the regular permutation flowshop scheduling problem referred to as the distributed permutation flowshop scheduling problem or (DPFSP). This new version assumes that there are a total of F identical factories or shops, each one composed of m machines disposed in series. The available jobs have to be distributed among the different factories or shops and then a processing sequence has to be derived for the jobs assigned to each factory. The author characterized the DPFSP and proposed different mixed integer linear programming models to solve the problem. They also discussed several heuristics based on dispatching rules, effective constructive heuristics and variable neighborhood descent methods. This problem has attracted considerable interest over the last few years. Many authors have presented new optimization methods to solve this problem. For example, Bargaoui et al. [26] used an artificial chemical reaction metaheuristic which objective is to minimize the maximum completion time to solve the distributed permutation flowshop scheduling problem with makespan criterion. In the proposed metaheuristic, the NEH heuristic was adapted to generate the initial population of molecules. Furthermore, a One-Point crossover and a greedy strategy were embedded in the algorithm in order to ameliorate the solution quality. Ruiz et al. [27] proposed Iterated Greedy algorithms based on improved initialization, construction and destruction procedures, along with a local search procedure.

Hatemi et al. [28] presented for the first time a combination of DPFSP and the Assembly Flow Shop Scheduling Problem referred to distributed assembly permutation flowshop scheduling problem (DAPFSP). DAPFSP contains two phase, production and assembly. At the first phase, it is to produce manufacture parts, just like in the regular DPFSP. The second phase is to assemble parts to make products in one assembly factory with only an assembly machine. The authors proposed two simple constructive algorithms to solve the problem. They also provided two sets of instances to test the model and algorithms.

The literature about the distributed permutation flowshop Problem is relatively small. Li et al. [29] addressed the same problem. They proposed four versions of genetic algorithm by combining the classical genetic algorithm with enhanced crossover and several local searches to solve the problem. Recently, Hatemi et al. [30] addressed the same problem with the additional consideration of sequence dependent setup times at both production and assembly stages. They proposed two simple heuristics and two metaheuristics to solve it. The performance of their proposed methods was compared through extensive computational and statistical experiments study. A hybrid biogeography-based optimization algorithm that integrates several novel heuristics is proposed by Lin and Zhang[31] to solve the DAPFSP with the objective of minimizing the makespan. The performance of their approach was evaluated based on two sets of benchmark instances issued from the literature. Compared to the best-known results, new best solutions for 71 small-sized instances and 91 large-sized instances were found.

To the best of our knowledge, Sang et al. [32] addressed in 2019 the first paper attempt to minimize total flowtime for the distributed assembly permutation flowshop scheduling problem. They proposed three variants of the discrete invasive weed optimization approach to solve the problem. To test the proposed algorithms, they carried out a comprehensive experiment based on 810 instances issued from literature. Numerical results and statistical analysis show that the presented algorithms perform substantially better than the other algorithms in for solving the DAPFSP with the total flowtime criterion.

For a complete review of the literature of the different variants of deterministic assembly scheduling problems, reader can refer the excellent work of Framinan et al. [33]. The authors reviewed a large number of papers in order to provide a comprehensive overview on scheduling with assembly operations. They proposed a unified notation for assembly scheduling models that encompass all concurrent-type scheduling problems. Using this notation, the existing contributions are reviewed and classified into a single framework, so a comprehensive, unified picture of the field is addressed.

1.3. Structure of the paper

Based on the studies aforementioned, we propose, for the variant of the flowshop analyzed in this paper: a MIP model; heuristics based on the algorithms proposed by Johnson [1] and by Nawaz et al. [10]; and GRASP and simulated annealing metaheuristics. We report computational experiments on random instances generated according to procedures used to generate benchmark instances found in the literature for the permutation flowshop. This work is an extension of the preliminary results in [34]. The paper is organized as follows. In Section 2 the mathematical model is presented. The proposed heuristics and metaheuristics are presented in Section 3 and Section 4, respectively. Computational experiments are reported in Section 5, and concluding remarks drawn in Section 6.

2. Mathematical formulation

We propose a MIP formulation based on the previous works developed by Wagner [35] and by Stafford [36]. According to the study conducted by Tseng et al. [4], such kind of formulation is the best for permutation flowshop. Let n be he number of jobs, and m_l be the number of machines in semi-line $l = 1, 2$ (before the synchronizing operation). In our model, a unique permutation has to be chosen for two flowshop problems with $m_l + 1$ machines each subject to the additional constraint that the completion times of each job on machines $m_1 + 1$ and $m_2 + 1$ are the same. Let p^l_{ki} be the processing time of job i on machine k of the problem l ($p_{(m_1+1)i} = p_{(m_2+1)i}$). The model can be generalized for an arbitrary number L of semi-lines. Thus, assuming, without loss of generality, $l = 1, 2$ flowshop problems with $m_l + 1$ machines each, the variables are defined as follows:

z_{ij} binary variable specifies if job i is assigned to the j^{th} position of the permutation (common two both $l = 1, 2$ flowshop problems);

x^l_{jk} idle time on machine k of the problem l before the job starts in the j^{th} position of the permutation;

y^l_{jk} idle time of job in the j^{th} position of the permutation after finishing the processing on machine k of problem l, while waiting for the machine $k+1$ of problem l to become available;

C^l_j completion time in problem l of the job in the j^{th} position of the permutation.

The makespan is given by the completion time of the job in the last position of the permutation. The model is written as follows:

$$\min C^1_n \tag{1}$$

$$\sum_{j=1}^{n} z_{ij} = 1;\ i = 1, \ldots, n \tag{2}$$

$$\sum_{i=1}^{n} z_{ij} = 1;\ j = 1, \ldots, n \tag{3}$$

$$\sum_{i=1}^{n} p^l_{ri} z_{ij+1} + y^l_{j+1r} + x^l_{j+1r} = y^l_{jr} + \sum_{i=1}^{n} p^l_{r+1i} z_{ij} + x^l_{j+1r+1};\ l = 1, 2, j = 1, \ldots, n-1; r = 1, \ldots, m_l \tag{4}$$

$$\sum_{r=1}^{k-1} \sum_{i=1}^{n} p^l_{ri} z_{i1} = x^l_{1k};\ l = 1, 2, k = 2, \ldots, m_l \tag{5}$$

$$\sum_{r=1}^{m_v} \sum_{i=1}^{n} p^v_{ri} z_{i1} \leq x^l_{1m_l+1};\ l, v = 1, 2 \tag{6}$$

$$y^l_{1k} = 0;\ l = 1, 2, k = 1, \ldots, m_l - 1 \tag{7}$$

$$x^l_{1m_l+1} - \left(x^l_{1m_l} + \sum_{i=1}^{n} p_{m_l i} z_{i1} \right) = y^l_{1m_l};\ l = 1, 2 \tag{8}$$

$$\sum_{u=1}^{j}\sum_{i=1}^{n} p^l_{m_l+1i} z_{iu} + \sum_{u=1}^{j} x^l_{um_l+1} = C^l_j;\ l = 1,2, j = 1,\ldots,n \tag{9}$$

$$C^1_j = C^2_j;\ j = 1,\ldots,n \tag{10}$$

$$z_{ij} \in \{0,1\};\ j = 1,\ldots,n, i = 1,\ldots,n \tag{11}$$

$$y^l_{jk}, x^l_{jk} \geq 0;\ l = 1,2, j = 1,\ldots,n; k = 1,\ldots,m_l + 1 \tag{12}$$

The objective function (1) minimizes the makespan. Constraints (2) and (3) assign one job to exactly one position in the permutation. Constraint (4) is the so called job-adjacency, machine linkage constraint [37]. Constraint (4) ensures equal time-slices on adjacent machines for each pair of consecutive jobs in the sequence. A time-slice between the completion of job in position j on machine r and the start of job in position $j+1$ on machine $r+1$ is analysed. On the left side is computed the idle time on machine r before starting job in position $j+1$, its processing time, and the idle time of the job in case machine $r+1$ is not free. On the right side is computed the idle time of job in position j before starting in machine $r+1$, its processing time, and the idle time on machine $r+1$ while waiting for job in position $j+1$ to finish in machine r. The computations on each side must occur in the same time-slice. Constraint (5) computes, from the second machine on, the idle time in each machine of each semi-line while waiting for the first job. Constraint (6) ensures that the idle time on the synchronization machine waiting for the first job is equal to the larger total processing time on each semi-line. Constraint (7) ensures that there is no idle time for the job assigned to the first position in each machine of each semi-line, but the first job may wait to be processed on the synchronization machine, which is ensured by constraint (8). Constraints (9) and (10) ensure the synchronization. Constraints (11) and (12) impose the variation domain of the variables.

The differences that have to be introduced in the model with respect to the classical permutation flowshop are due to the fact that we have synchronize each problem l at machines m_{l+1} (the synchronization operation duplicated to be the last machine in each problem l). To do this we have to compute in (9) the completion time C_j^l for each job j in each flowshop problem l, instead of just the makespan, and impose them to be equal in (10).

This implies (i) that the m_{l+1} machine in problem l may have to wait for the first job in the sequence more than the sum of the processing times of such job on the previous machines (which is assumed to be equal in modeling the classical permutation flowshop). Thus, in (6) the waiting time on the m_{l+1} machine for the first job in each problem l must be greater or equal than the processing times in the previous machines of all problems, i.e., the two halves of a job must be completed before start the synchronization operation. And (ii) that the first job of the sequence may have to wait in problem l before starts its operation at the m_{l+1} machine (which is assumed to not occur in modeling the classical permutation flowshop). Indeed, we compute in (8) the waiting time of the fist job after being completed in the m_l machine in problem l because machine m_{l+1} has to be synchronized.

Let us illustrate with the example in Figure 2. The synchronization operation is the third machine in each problem l. We have $x_{1,3}^1 = 4$, which is greater than the time spent by job yellow to be processed in the first two machines of problem $l = 1$, since the third machine of problem $l = 1$ has to wait job yellow to be processed in the first two machines of problem $l = 2$. We also have $y_{1,2}^1 = 1$, which is the time job yellow has to wait before start processing at the third machine due to synchronization.

3. Heuristics

In this section, we propose adaptations of the NEH heuristic by Nawaz et al. [10] and also adaptations of the algorithm of Johnson [1] to obtain feasible solutions for the flowshop scheduling problem with parallel semi-lines and the

synchronization operation.

3.1. Adaptations of the NEH heuristic

The NEH heuristic for the classical permutation flowshop starts by sorting the n jobs in decreasing order of the sums of processing times on all the m machines. Then, a partial scheduling consists of the first two jobs of this order in a sequence that minimizes the makespan. The other jobs, from the third, are inserted (one at a time) in the position of the partial scheduling leading to the smallest makespan. The relative positions between jobs already inserted in the partial scheduling do not change. We develop three adaptations of the NEH algorithm : NEH_{av} - the average of the processing times p^1_{ki} and p^2_{ki} for each $k = 1, \dots m_l$ and for each job i, NEH_{hi} - the highest processing time between p^1_{ki} and p^2_{ki} for each $k = 1, \dots m_l$ and for each job i, and NEH_{sep} - where each semi-line is considered separately including the synchronization operation.

The NEH_{av} and the NEH_{hi} heuristics require the same number of machines in each semi-line, i.e, $m = m_1 = m_2$. Let f designate the final synchronization machine. The general principle is to reduce the two semi-lines to a single line and apply the NEH heuristic. We do this by replacing at each stage the corresponding machines in each semi-line for a a single machine, as illustrated in Figure 3.

In the NEH_{av} heuristic, for each job $j = 1, \dots, n$, we compute $\bar{p}_{kj} = (p^1_{kj} + p^2_{kj})/2$, where $k = 1, \dots, m$ is the k^{th}-machine in each semi-line. At this point we have an instance of the classical permutation flowshop with $m+1$ machines where, for each job j, $\bar{p}_{kj}$ is the processing time on machine $k = 1, \dots, m$ and p_{fj} is the processing time on the last machine, and apply the NEH heuristic to obtain a sequence seq_{av}. Finally, we compute the makespan incurred by the sequence seq_{av} in the whole system with the actual processing times p^l_{kj} for each semi-line $l = 1, 2$.

The NEH_{hi} heuristic works in a similar manner. For each job $j = 1, \dots, n$, we compute $\bar{p}_{kj} = \max\{p^1_{kj}, p^2_{kj}\}$, where $k = 1, \dots, m$ is the k^{th}-machine in each semi-line. Analogously to NEH_{av}, we obtain by the NEH heuristic a sequence

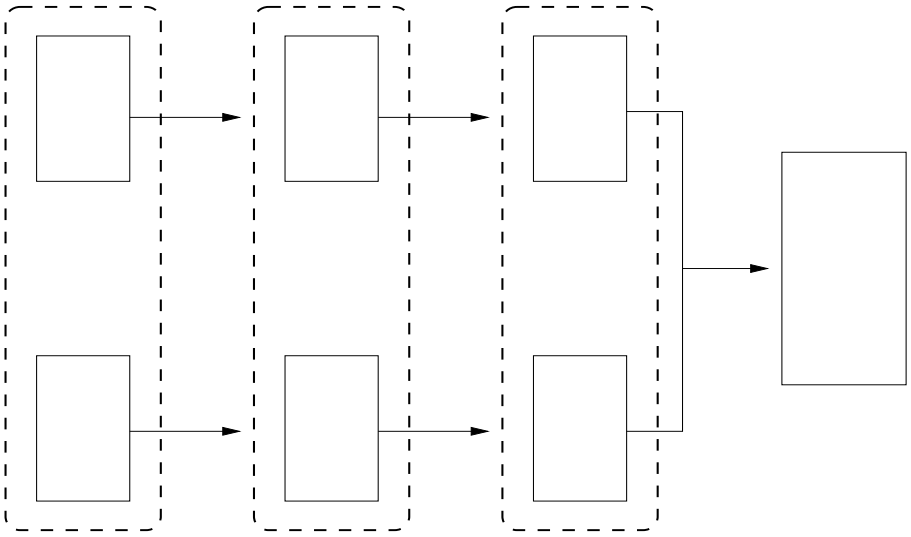


Figure 3: Reducing the semi-lines to a single line, $m = 3$.

seq_{hi} for an instance of the classical permutation flowshop with $m+1$ machines where, for each job j, $\bar{p}_{kj}$ is the processing time on machine $k = 1, \ldots, m$ and p_{fj} is the processing time on the last machine. We then compute the makespan incurred by the sequence seq_{hi} in the whole system with the actual processing times p^l_{kj} for each semi-line $l = 1, 2$.

In the NEH_{sep} heuristic we generate two instances of the classical permutation flowshop. NEH_{sep} does not require the same number of machines in each semi-line. Each instance is composed of the machines of one of the semi-lines along with the synchronization operation, as illustrated in Figure 4. We apply, to obtain a sequence $\text{seq}^l{}_{\text{sep}}$, the NEH heuristic to each instance $l = 1, 2$ with $m_l + 1$ machines where, for each job j, p^l_{kj} is the processing time on machine $k = 1, \ldots, m_l$ and p_{fj} is the processing time on the last machine. We adopt the sequence $\text{seq}^l{}_{\text{sep}}$, $l = 1, 2$, leading to the smallest makespan in the whole system with the two semi-lines and the synchronization operation.

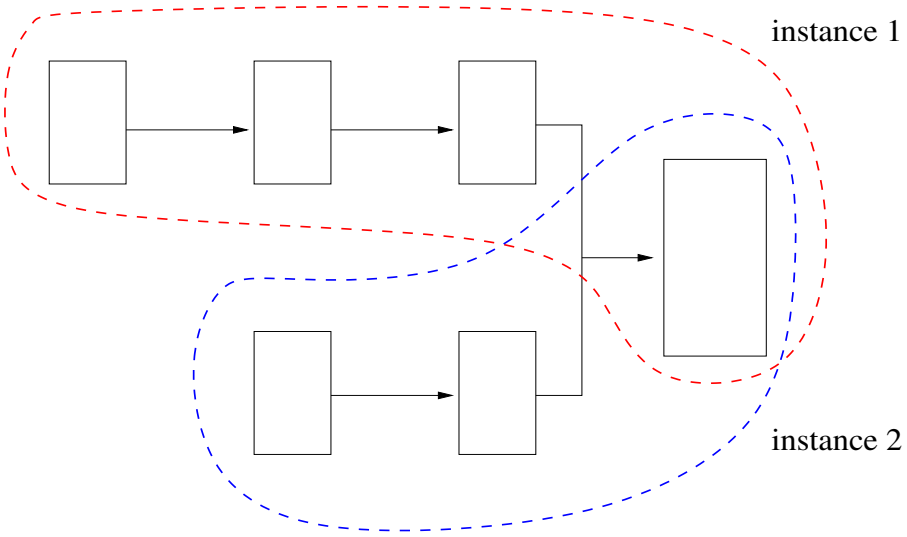


Figure 4: Generating two instances of the classical permutation flowshop.

3.2. Adaptations of Johnson's algorithm

Johnson's algorithm obtains a sequence that minimizes the makespan for the flowshop problem with two machines. The optimal sequence begins with the jobs having the processing time on the first machine smaller than the processing time on the second machine sorted in increasing order of processing times on the first machine, and ends with the remaining jobs in decreasing order of processing times on the second machine. We develop two adaptations of the Johnson's algorithm. The general principle is to consider the studied system as a flowshop with two machines by setting the synchronization operation as the second machine.

In the adaptation denoted by Joh_{av}, for each job $j = 1, \ldots, n$, the average processing time $\frac{\sum_{k=1}^{m_1} p_{kj}^1 + \sum_{k=1}^{m_2} p_{kj}^2}{m_1+m_2}$ in the machines of the two semi-lines is used as the processing time on the first machine. The processing time of the synchronization operation is the processing time on the second machine. We then apply Johnson's algorithm, and compute, for the sequence obtained, the makespan in the whole system with the actual processing times p_{kj}^l for each semi-line $l = 1, 2$. The adaptation denoted by Joh_{hi} works in a similar manner,

but the largest processing time $\max_{l=1,2,k=1,\ldots,m_l}\{p^l_{kj}\}$ among the machines of the semi-lines is used as the processing time of job j on the first machine.

4. Metaheuristics

We propose to use the previously described heuristics in the construction phase of a GRASP algorithm. GRASP was successfully applied to the permutation flowshop by Prabhaharan et al. [21], and to a multi-objective variant by Arroyo and de Souza Pereira [38]. Alternatively, we propose a simulated annealing algorithm, which was also successfully applied to the permutation flowshop by Hurkała and Hurkała [39], and to a multi-objective variant by Jarosław et al. [40].

4.1. GRASP

GRASP is a multistart metaheuristic, see, for instance, Resende and Ribeiro [41]. A GRASP iteration consists basically of two phases: a construction phase that builds a feasible solution using a randomized greedy heuristic, followed by a local search phase. We propose two versions of GRASP to try to find optimal or near-optimal solutions for the flowshop problem with parallel semi-lines and the synchronization operation. These variants differ in the heuristic employed in the construction phase. One version uses NEH_{sep} and the other uses Joh_{av}, since these were the procedures to obtain the best results in our computational experiments, c.f., Section 5.

Figure 5 presents the pseudo-code of the construction phase of GRASP_NEH_{sep}. The procedure is written for a number L of semi-lines. The main loop in lines 1 to 10 treats, as NEH_{sep}, each semi-line l along with the synchronization operation as an independent instance of the classical permutation flowshop. In line 2, for each job j, p_j is the sum of processing times on all the $m_l + 1$ machines. The inner loop in lines 3 to 8 applies a randomized version of NEH_{sep}. Instead of taking each time a job in decreasing order of p_j, a job q is randomly chosen from the Restricted Candidate List RCL_l. Given a percentage α of the total

number n of jobs, at each time, RCL_l contains the αn jobs with the largest p_j not yet added to the sequence (if it remains lesser jobs than αn, then all remaining jobs are inserted at RCL_l). The insertion of job q randomly selected from RCL_l in the sequence under construction is done according to the NEH heuristic. In line 9, the makespan mk^l incurred by seq^l in the whole system with the two semi-lines and the synchronization operation is computed. At each GRASP iteration the construction phase returns, in line 11, the sequence seq^l , $l = 1, \ldots, L$, leading to the smallest makespan.

Procedure `Construction Phase GRASP_NEH`$_{\text{sep}}$

1 **For** $l = 1$ **to** L **do**
2 Let J be the set of n jobs, and compute $p_j = \sum_{k=1}^{m_l} p_{kj}^l + p_{fj}$ for each job $j \in J$.
3 **For** $t = 1$ **to** n **do**
4 Let $\text{RCL}_l \subseteq J$ be the set of the $\min\{\alpha n, n - t + 1\}$ jobs with the largest p_j.
5 Take at random a job $q \in \text{RCL}_l$.
6 Insert q in the best of the t possible positions in the partial sequence seq^l
7 Let $J = J - \{q\}$
8 **End-For**
9 Compute be the makespan mk^l incurred by seq^l in the whole system.
10 **End-For**
11 Return the sequence seq^l, $l = 1, \ldots, L$, leading to the smallest mk^l.

End-Procedure

Figure 5: Pseudo-code of the construction phase of $\text{GRASP\_NEH}_{\text{sep}}$.

Figure 6 presents the pseudo-code of the construction phase of $\text{GRASP\_Joh}_{\text{av}}$. The procedure uses two Restricted Candidate Lists: RCL_u for the upward part, and RCL_d for the downward part of the sequence seq^{Jav}. In line 1, for each job j, p_j is the average processing time on all the machines of the L semi-lines (the processing time of job j reducing the semi-lines to the first machine, the synchronization operation being the second machine). As in Johnson's algorithm, in line 2 the jobs are partitioned in two subsets J_u and J_d by comparing p_j to p_{fj}. Note that n_u and n_d are set in line 3, and do not change along the procedure. The first loop in lines 4 to 9 builds with a randomized version of

Johnson's algorithm the upward part of seq$^{\text{Jav}}$. Instead of taking each time a job in increasing order of p_j, a job q is randomly chosen from RCL_u. Given a percentage α of n_u, at each time, RCL_u contains the αn_u jobs with the smallest p_j not yet added to the sequence (if it remains lesser jobs than αn_u, then all remaining jobs in J_u are inserted at RCL_u). As in Johnson's algorithm, job q is inserted in the last position of seq$^{\text{Jav}}$. The second loop in lines 10 to 14 builds in an analogous manner the downward part of seq$^{\text{Jav}}$, as each time a job q is randomly chosen from RCL_d which contains the αn_d jobs with the largest p_j not yet added to the sequence. The makespan incurred by seq$^{\text{Jav}}$ in the whole system with the two semi-lines and the synchronization operation is computed in line 16, and seq$^{\text{Jav}}$ is returned by the construction phase at each GRASP iteration in line 17.

Procedure `Construction Phase GRASP_Joh`$_{\text{av}}$

1 Let J be the set of n jobs, and compute $p_j = \frac{\sum_{l=1}^{L}\sum_{k=1}^{m_l} p_{kj}^l}{\sum_{l=1}^{L} m_l}$ for each job $j \in J$.
2 Let $J_u \subseteq J$ (resp. $J_d \subseteq J$) be the set of jobs such that $p_j \leq p_{fj}$ (resp. $p_j > p_{fj}$).
3 Set $n_u = |J_u|$ and $n_d = |J_d|$.
4 **For** $t = 1$ **to** n_u **do**
5 Let $\text{RCL}_u \subseteq J_u$ be the set of the $\min\{\alpha n_u, n_u - t + 1\}$ jobs with the smallest p_j.
6 Take at random a job $q \in \text{RCL}_u$.
7 Insert q in the t-th position in the partial sequence seq$^{\text{Jav}}$
8 Let $J_u = J_u - \{q\}$
9 **End-For**
10 **For** $t = 1$ **to** n_d **do**
11 Let $\text{RCL}_d \subseteq J_d$ be the set of the $\min\{\alpha n_d, n_d - t + 1\}$ jobs with the largest p_j.
12 Take at random a job $q \in \text{RCL}_d$.
13 Insert q in the $(n_u + t)$-th position in the partial sequence seq$^{\text{Jav}}$
14 Let $J_d = J_d - \{q\}$
15 **End-For**
16 Compute be the makespan incurred by seq$^{\text{Jav}}$ in the whole system.
17 Return the sequence seq$^{\text{Jav}}$.

End-Procedure

Figure 6: Pseudo-code of the construction phase of GRASP_Joh$_{\text{av}}$.

The local search for both versions of GRASP is based on swap moves. Let π be a permutation of the n jobs. The local search procedure evaluates all the possible $O(n^2)$ moves swapping pairs of jobs at two different positions in π. Initially, π is set as the permutation returned by the construction phase. The local search uses the strategy of best improvement move. Let π' be the solution with lowest cost in the neighborhood of π. If the cost of π' is lower than the cost of π, then π is updated to π' and the search resumes. Otherwise π is returned as the local optimum.

4.2. Simulated annealing

We also use NEH_{sep} and Joh_{av} as initial solutions to a Simulated Annealing (SA) algorithm. Given a current solution s, SA proceeds generating at each iteration a neighbor solution s' with a swap move at random. If s' improves the makespan of s, then the current solution is updated. To prevent getting stuck in a local optima, the algorithm allows some worsening solutions. This is done by respecting a probability of allowance in relation to a temperature T. Let Δ be the difference in the makespan between s' and s. The worsening solution is accepted if a randomly chosen value between 0 and 1 is lower than $e^{-\Delta/T}$. The algorithm stopping criterion is determined by the slow cooling of the initial temperature. After a number of iterations with the same temperature without improvement, the temperature is updated to αT, $\alpha \in (0, 1)$. Note that GRASP and SA exploit the same neighborhood. At the end of SA, as an intensification strategy, we apply the local search based on swap moves to the solution returned by SA.

5. Computational experiments

The computational experiments were structured into three comparative settings: comparison of the heuristics proposed in Section 3 in terms of solution quality, tuning parameters of the metaheuristics proposed in Section 4, and evaluation of the effectiveness of the best metaheuristic configurations with respect to optimal or lower bounds obtained with the MIP model proposed in Section 2.

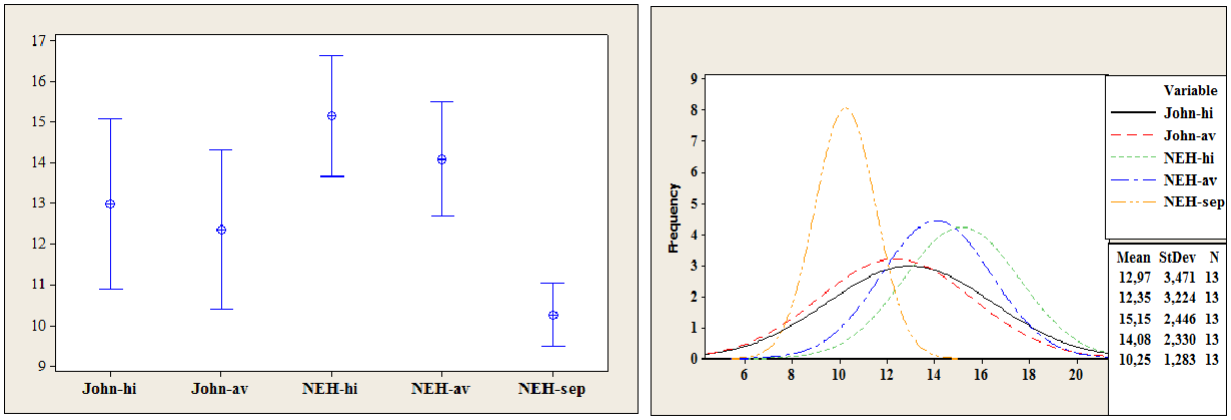
For such purpose, we generated two set of instances according to the guidelines introduced by Taillard [42]. In the first set of instances the semi-lines have the same number of machines each, while in the second set they have different number of machines. A total of 230 instances were generated with 10, 20, and 50 jobs, and 3, 5, 7, and 11 machines, including the final synchronization machine. For example, an instance with 3 machines belonging to the first set has 2 machines in each semi-line, and the final synchronization machine, whereas an instance with 3 and 5 machines belonging to second the set has 2 machines in one semi-line, 4 machines in the other semi-line, and the final synchronization machine. The set with the same number of machines contains 120 instances: 10 instances for each combination number of jobs - number of machines. The set with different number of machines contains 110 instances: 50 instances with 10 jobs, 50 instances with 20 jobs, and 10 instances with 50 jobs.

The computational experiments were run on a Intel Core i3, 3.1GHz with 4GB of RAM. The results of the MIP model were obtained with CPLEX 12.6.1, and the proposed algorithms were implemented in C++. We were able to obtain optimal solutions for 186 instances.

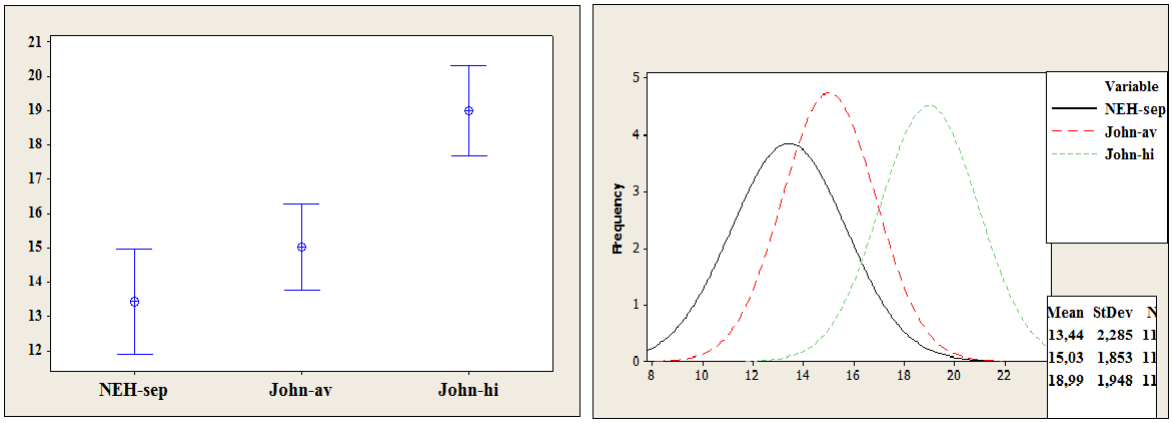
In the first two comparative settings, i.e., comparisons of the heuristics proposed in Section 3 and tuning parameters of the metaheuristics proposed in Section 4, the quality of the solution obtained by the proposed algorithms is measured by the percentage gap $= \frac{ub-opt}{opt} * 100$, where ub is the solution obtained by the proposed algorithm and opt is the optimal or the best solution obtained in these settings (in the case of the remaining open instances).

Figure 7 shows, for each heuristic proposed in Section 3, the average percentage gap, the range interval of the average gaps, and the fit to a normal distribution. Figure 7a shows results for the instances with the same number number of machines in each semi-line, and Figure 7b results for the instances with different number of machines.

Considering the results shown in Figure 7a, we can see that the idea of reducing the problem to a classical permutation flowshop is not effective, since NEH_{av} and NEH_{hi} present the higher gaps. The performance of the adaptations



(a) Same number of machines.



(b) Different number of machines.

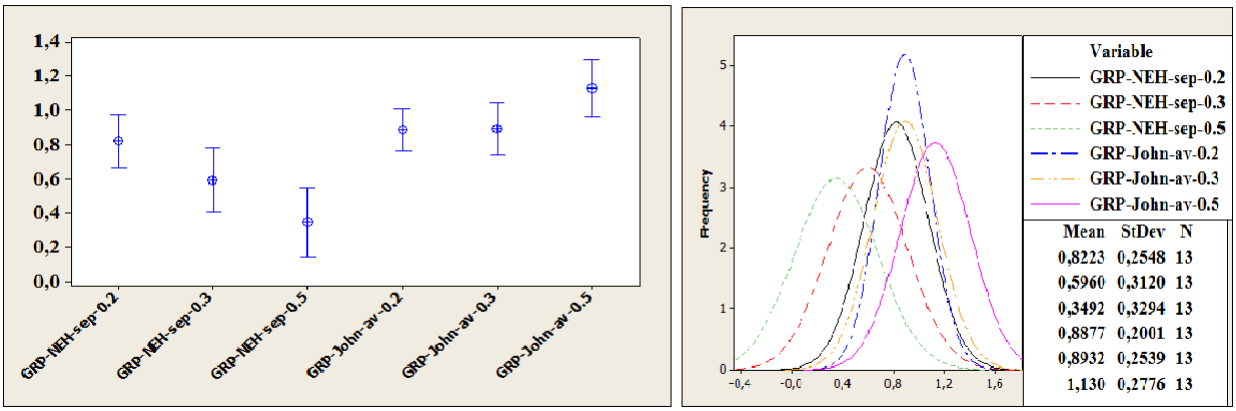
Figure 7: Comparison of the proposed heuristics.

of Johnson's algorithm vary considerably, since Joh_{av} and Joh_{hi} present the higher standard deviations. The heuristic NEH_{sep} obtained the best average results on the both set of instances. We remark that for the instances for which the optimal solution were obtained, the optimal values are higher that the optimal values considering each semi-line along with the synchronization machine as a classical permutation flowshop, i.e., there is not a semi-line that dominates the other one. The average gaps observed for NEH_{sep}, Joh_{av}, and Joh_{hi} are higher for the set with different number of machines. As NEH_{sep} and Joh_{av} were the heuristics to present the best results, they were used in our GRASP and SA algorithms, c.f., Section 4.

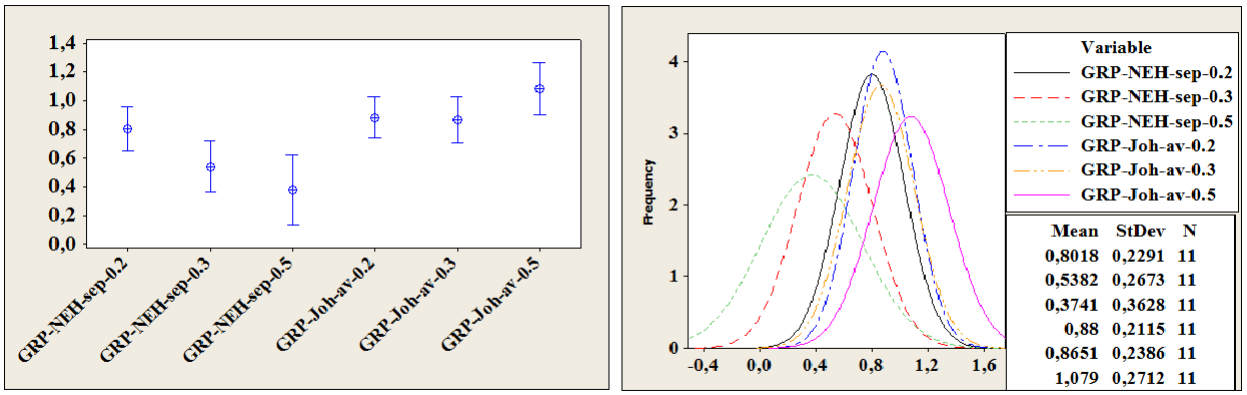
Figure 8 shows results for GRASP with different values of the parameter α which controls the cardinality of the RCL. The number of GRASP iterations without improvement, i.e., the stopping criteria, was set to 100. Increasing the value of α leads to a RCL with a larger cardinality. We consider for α the values 20%, 30%, and 50%. We were able to drastically reduce solution costs embedding NEH_{sep} and Joh_{av} into GRASP. GRASP_NEH_{sep} with $\alpha = 50\%$ presents average gaps of less than 0.5% to the optimal or to the best solution obtained in this setting.

Figure 9 shows results for SA with different values of the parameter α, the cooling factor, which controls the reduction of the temperature. The initial and the final temperatures were set to 6000 and 10^{-4}, respectively. The number of SA iterations with a constant temperature without improvement was set to 90. We consider for α the values 0.20, 0.50, and 0.95. Although important improvements can be observed, SA was not able to improve average gaps in the same manner as GRASP. The best results with SA presents average gaps higher than 2%.

We now report in a final comparative setting detailed results to assess the effectiveness of GRASP. For such purpose we report optimality gaps computed with respect to optimal or best lower bounds obtained with the MIP model proposed in Section 2. Since GRASP_NEH_{sep} with $\alpha = 50\%$ obtained the best results in the previous experiments, we investigate greater values of $\alpha = 75\%$,

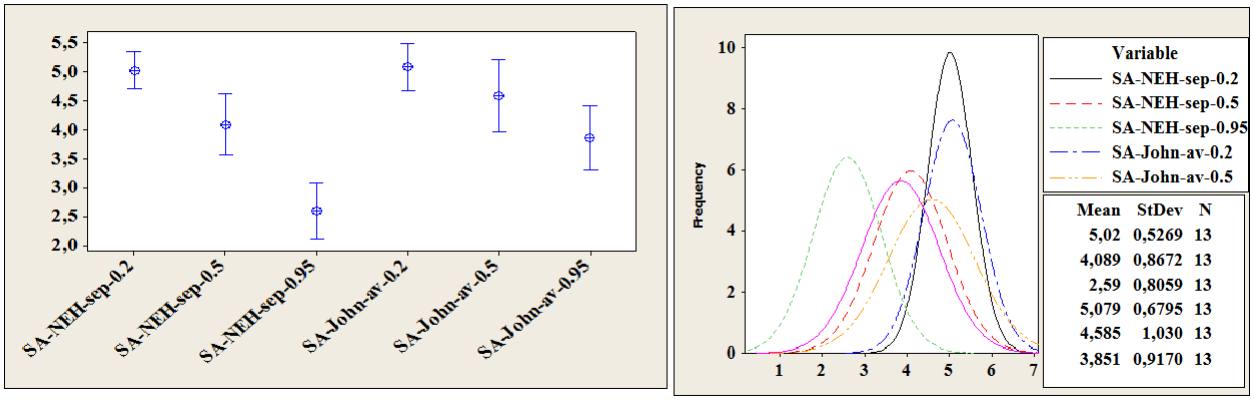


(a) Same number of machines.

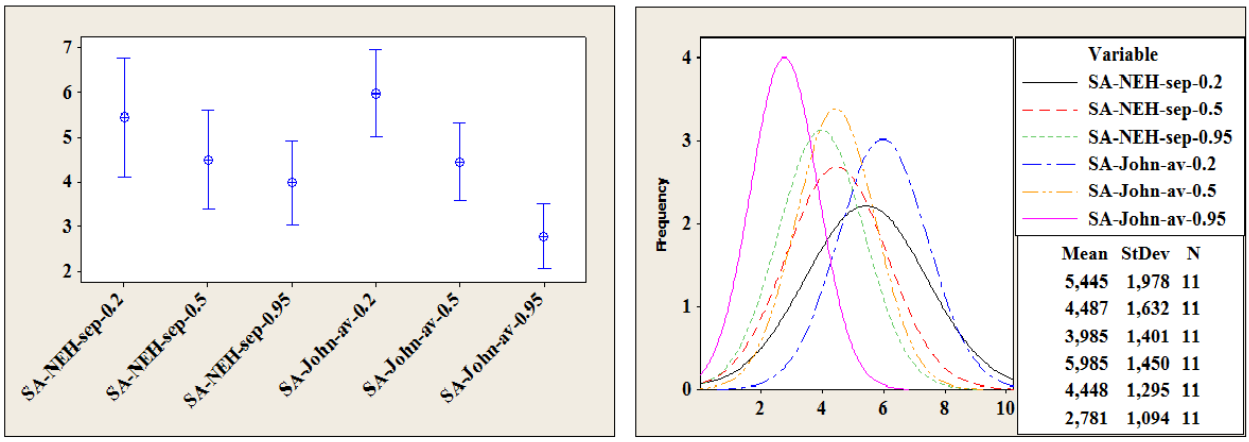


(b) Different number of machines.

Figure 8: GRASP performances regarding different values of the Restricted Candidate List (RCL) parameter.



(a) Same number of machines.



(b) Different number of machines.

Figure 9: SA performances regarding different the values of the cooling factor parameter.

and $\alpha = 100\%$.[1] It turns out that best results were obtained with $\alpha = 100\%$, which corresponds to a random multistart algorithm, i.e., each iteration consists of generating a completely random permutation and applying the local search procedure.

Tables 2 and 3 present average results for GRASP_NEH$_{sep}$ with $\alpha = 50\%$, $\alpha = 75\%$, and $\alpha = 100\%$ on instances with the same and different number of machines in each semi-line, respectively. Each line of the tables corresponds to average results for 10 instances of a given combination of number of jobs - number of machines. The first column indicates the number of jobs and the number of machines. For example, in Table 2, $E10 \times 03$ indicates instances with 10 jobs, 2 machines in each semi-line, and the final synchronization machine, whereas in Table 3, $D10 \times 03 \times 05$ indicates instances with 10 jobs, 2 machines in one semi-line, 4 machines in the other semi-line, and the final synchronization machine. For the MIP model it is shown, in the second and third columns, the number of instances for each combination solved to optimality and the average computational time in seconds, respectively. Then, for each value of α, it is shown the number of instances for each combination the optimal solution was found, the average optimality gap in percentage, the average computational time in seconds, and the average total number of iterations.

Note that not only the number of jobs, but also the number of machines play an important role in how difficult is to solve the problem to optimality with the MIP model. For example, instances with 50 jobs and 3 machines in each semi-line were solved 10 times faster in average than instances with 20 jobs and 5 machines in each semi-line. Instances with a large number of jobs and machines were out of reach for the MIP model. Results show that GRASP_NEH$_{sep}$ is an effective and robust algorithm to tackle the problem. The overall average optimality gaps are smaller than 1% for all three values of α, and only for 4 combinations with 11 machines out of 23 combinations of number of jobs

[1] We thank the anonymous reviewer for his/her suggestion to investigate greater values of α.

- number of machines average optimality gaps are in the range between 1% and 5%. In particular GRASP_NEH$_{sep}$ with $\alpha = 100\%$, the random multistart version, obtained the optimal solution for 186 out of 230 instances and for only 2 combinations the average optimality gap exceeded 1% ($E50 \times 11$ with 2.43% and $D20 \times 03 \times 11$ with 1.14%). Moreover, these results were obtained with low computational times, even for instances out of reach for the MIP model.

Tables 4 to 9 present detailed results for the combinations which optimal values were not obtained for most instances, namely: $E20 \times 07$, $E20 \times 11$, $E50 \times 07$, $E50 \times 11$, $D20 \times 03 \times 11$, and $D20 \times 05 \times 11$, in this order. In these tables, the first column identifies the instance, and the second column presents the best lower bound obtained when running the MIP model. Then, for each value of α, it is shown for each instance the upper bound obtained, the optimality gap in percentage, the computational time in seconds, and the total number of iterations. The detailed results on the harder instances confirm that GRASP_NEH$_{sep}$ is an effective and robust algorithm to tackle the problem, specially GRASP_NEH$_{sep}$ with $\alpha = 100\%$ that obtained optimality gaps below 5% for all instances but $E50 \times 11 - 6$.

6. Conclusion

This work focused on the development of optimization methods to solve a variant of the permutation flowshop scheduling problem with parallel semi-lines and a final synchronization operation. This study emerged from a practical situation in a welding process of an industry that manufactures electro-electronic system products, and may be a common problem in other industrial sectors as well. Our purpose was to design efficient methods to solve the problem, which, to the best of our knowledge, has not yet been treated.

In a first approach, a mixed-integer linear programming model, inspired from the classical permutation flowshop, was introduced. This model required adaptations regarding to the treatment of the semi-lines separately and a particular

attention in the finalization of the semi-lines since this is the point of junction of the two halves of the jobs that are produced in the two semi-lines. The model was efficient for small instances, but for larger instances, it was not able to find optimal solutions in moderate computational times. We also proposed constructive heuristics and methaheuristics in an attempt to find optimal or near-optimal solutions with reasonable computational times. In particular, an adaptation of the NEH heuristic embedded in a GRASP algorithm lead to an effective and robust algorithm to tackle the problem, obtaining average optimality gaps of less than 1% in low computational times.

Future extension of our work could focus on further exploration of problem-specific characteristics and developing more effective methods and local search procedures for this problem. Moreover, due to the effectiveness of the metaheuristic methods, it could be interesting to analyze their performances by using other objective criteria such as the total flowtime or total tardiness.

	MIP model		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	opt	t(s)	opt	g(%)	t(s)	it	opt	g(%)	t(s)	it	opt	g(%)	t(s)	it
$E10 \times 03$	10	1	6	0.09	27	105	10	0.00	35	110	10	0.00	30	111
$E10 \times 05$	10	1	6	0.11	34	118	10	0.00	38	116	10	0.00	30	111
$E10 \times 07$	10	2	6	0.12	48	117	10	0.00	50	114	10	0.00	48	110
$E10 \times 11$	10	11	3	0.27	46	112	8	0.02	52	112	10	0.00	49	112
$E20 \times 03$	10	1	5	0.09	44	110	10	0.00	48	112	10	0.00	49	110
$E20 \times 05$	10	405	5	0.12	44	112	7	0.05	45	122	10	0.00	49	119
$E20 \times 07$	4	2289	4	0.14	43	132	5	0.23	51	139	4	0.14	48	136
$E20 \times 11$	0	7542	0	2.29	43	143	0	1.29	52	146	0	0.93	47	139
$E50 \times 03$	10	6	5	0.11	46	106	8	0.01	51	140	10	0.00	49	148
$E50 \times 05$	9	4916	4	0.18	47	152	7	0.05	51	145	9	0.02	47	149
$E50 \times 07$	4	13695	0	0.58	57	149	3	0.13	59	160	5	0.19	60	141
$E50 \times 11$	0	12746	0	3.39	57	166	0	4.08	61	174	0	2.43	60	165
av gap(%)				0.62				0.49				0.31		

Table 2: Average results for GRASP_NEH$_{sep}$ with $\alpha = 50\%$, $\alpha = 75\%$ and $\alpha = 100\%$ - same number of machines.

	MIP model		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	opt	t(s)	opt	g(%)	t(s)	it	opt	g(%)	t(s)	it	opt	g(%)	t(s)	it
$D10 \times 03 \times 05$	10	1	6	0.07	27	108	8	0.03	27	112	10	0.00	26	118
$D10 \times 03 \times 07$	10	1	5	0.54	33	120	7	0.05	33	119	10	0.00	33	119
$D10 \times 03 \times 11$	10	1	6	0.36	48	121	7	0.06	45	122	10	0.00	46	121
$D10 \times 05 \times 07$	10	1	6	0.14	45	128	9	0.01	45	122	10	0.00	48	127
$D10 \times 05 \times 11$	10	2	5	0.06	46	126	7	0.03	45	125	10	0.00	52	122
$D20 \times 03 \times 05$	10	4	5	0.23	48	121	8	0.08	49	126	10	0.00	53	129
$D20 \times 03 \times 07$	10	53	4	0.32	44	127	7	0.05	49	130	10	0.00	54	132
$D20 \times 03 \times 11$	5	14250	0	1.88	50	140	1	0.62	47	138	4	1.14	52	138
$D20 \times 05 \times 07$	10	200	3	0.14	49	135	7	0.04	48	137	10	0.00	52	137
$D20 \times 05 \times 11$	4	18208	0	1.24	51	145	0	0.62	48	146	4	0.64	52	166
$D50 \times 03 \times 05$	10	154	4	0.08	58	156	9	0.01	50	149	10	0.00	53	164
av gap(%)				0.46				0.14				0.16		

Table 3: Average results for GRASP_NEH$_{sep}$ with $\alpha = 50\%$, $\alpha = 75\%$ and $\alpha = 100\%$ - different number of machines.

		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	LB	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it
$E20 \times 07 - 1$	642	643	0.16	43	115	645	0.47	52	136	643	0.16	53	120
$E20 \times 07 - 2$	946	946	0.00	44	160	946	0.00	53	150	946	0.00	50	126
$E20 \times 07 - 3$	1284	1284	0.00	42	131	1284	0.00	53	122	1284	0.00	43	134
$E20 \times 07 - 4$	725	725	0.00	42	129	725	0.00	43	146	725	0.00	45	145
$E20 \times 07 - 5$	1063	1066	0.28	48	138	1070	0.66	46	125	1070	0.66	49	139
$E20 \times 07 - 6$	1357	1362	0.37	42	133	1369	0.88	48	155	1359	0.15	47	145
$E20 \times 07 - 7$	1357	1362	0.37	45	132	1360	0.22	55	165	1360	0.22	49	125
$E20 \times 07 - 8$	1361	1363	0.15	42	136	1362	0.07	62	128	1362	0.07	51	149
$E20 \times 07 - 9$	1342	1343	0.07	44	138	1342	0.00	54	124	1344	0.15	49	150
$E20 \times 07 - 10$	690	690	0.00	43	113	690	0.00	48	136	690	0.00	49	125

Table 4: Detailed results for GRASP with $\alpha = 50\%$, $\alpha = 75\%$, and $\alpha = 100\%$ on instances E20×07.

		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	LB	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it
$E20 \times 11 - 1$	805	809	0.50	44	150	812	0.87	50	143	808	0.37	48	141
$E20 \times 11 - 2$	1601	1608	0.44	45	135	1606	0.31	54	147	1604	0.19	43	133
$E20 \times 11 - 3$	1375	1376	0.07	42	115	1382	0.44	50	118	1380	0.36	46	141
$E20 \times 11 - 4$	1229	1296	5.45	44	157	1234	0.41	49	155	1234	0.41	46	156
$E20 \times 11 - 5$	926	1018	9.93	46	112	929	0.32	53	160	930	0.43	46	149
$E20 \times 11 - 6$	1687	1706	1.13	40	155	1700	0.77	50	124	1700	0.77	44	139
$E20 \times 11 - 7$	1593	1600	0.44	40	117	1600	0.44	54	165	1600	0.44	44	145
$E20 \times 11 - 8$	1581	1608	1.71	43	169	1600	1.20	54	174	1606	1.58	56	113
$E20 \times 11 - 9$	1599	1638	2.44	44	132	1639	2.50	55	137	1635	2.25	51	126
$E20 \times 11 - 10$	1600	1613	0.81	43	184	1690	5.62	49	133	1640	2.50	47	148

Table 5: Detailed results for GRASP with $\alpha = 50\%$, $\alpha = 75\%$, and $\alpha = 100\%$ on instances E20×11.

		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	LB	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it
$E50 \times 07 - 1$	2081	2128	2.26	57	166	2083	0.10	58	172	2092	0.53	59	129
$E50 \times 07 - 2$	1455	1460	0.34	55	166	1456	0.07	57	170	1455	0.00	58	144
$E50 \times 07 - 3$	945	946	0.11	58	124	945	0.00	59	139	945	0.00	61	165
$E50 \times 07 - 4$	1603	1605	0.12	58	127	1604	0.06	59	164	1603	0.00	57	149
$E50 \times 07 - 5$	2064	2066	0.10	57	133	2064	0.00	59	139	2064	0.00	62	122
$E50 \times 07 - 6$	2185	2186	0.05	57	150	2186	0.05	57	183	2186	0.05	64	137
$E50 \times 07 - 7$	1475	1476	0.07	58	141	1475	0.00	60	175	1475	0.00	61	143
$E50 \times 07 - 8$	2224	2249	1.12	57	133	2238	0.63	59	173	2238	0.63	60	154
$E50 \times 07 - 9$	1575	1600	1.59	58	180	1580	0.32	60	159	1582	0.44	61	127
$E50 \times 07 - 10$	1643	1644	0.06	57	170	1645	0.12	60	125	1648	0.30	60	138

Table 6: Detailed results for GRASP with $\alpha = 50\%$, $\alpha = 75\%$, and $\alpha = 100\%$ on instances E50×07.

		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	LB	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it
$E50 \times 11 - 1$	2437	2548	4.55	54	183	2582	5.95	64	150	2518	3.32	49	172
$E50 \times 11 - 2$	2596	2760	6.32	59	189	2624	1.08	62	181	2664	2.62	54	164
$E50 \times 11 - 3$	2590	2662	2.78	59	153	2778	7.26	60	153	2606	0.62	58	165
$E50 \times 11 - 4$	2705	2736	1.15	59	162	2736	1.15	61	162	2736	1.15	61	176
$E50 \times 11 - 5$	2613	2802	7.23	56	184	2800	7.16	61	176	2700	3.33	60	185
$E50 \times 11 - 6$	3198	3225	0.84	57	156	3456	8.07	63	183	3358	5.00	63	171
$E50 \times 11 - 7$	2554	2557	0.12	57	152	2556	0.08	60	183	2556	0.08	66	162
$E50 \times 11 - 8$	2412	2549	5.68	56	155	2514	4.23	60	187	2514	4.23	63	164
$E50 \times 11 - 9$	2984	3060	2.55	58	164	3022	1.27	65	175	3022	1.27	62	167
$E50 \times 11 - 10$	3067	3150	2.71	58	167	3208	4.60	59	189	3150	2.71	63	128

Table 7: Detailed results for GRASP with $\alpha = 50\%$, $\alpha = 75\%$, and $\alpha = 100\%$ on instances E50×11.

		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	LB	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it
$D20 \times 03 \times 11 - 1$	726	768	5.78	46	144	734	1.10	46	131	738	1.65	56	128
$D20 \times 03 \times 11 - 2$	1122	1125	0.27	46	156	1124	0.18	45	139	1124	0.18	57	119
$D20 \times 03 \times 11 - 3$	1516	1522	0.40	48	145	1518	0.13	48	118	1516	0.00	51	153
$D20 \times 03 \times 11 - 4$	836	844	0.96	56	150	837	0.12	44	122	836	0.00	51	148
$D20 \times 03 \times 11 - 5$	1171	1182	0.94	51	118	1190	1.62	48	155	1224	4.53	51	137
$D20 \times 03 \times 11 - 6$	775	785	1.29	51	126	776	0.13	48	134	775	0.00	49	139
$D20 \times 03 \times 11 - 7$	804	840	4.48	50	131	814	1.24	45	140	816	1.49	49	136
$D20 \times 03 \times 11 - 8$	813	826	1.60	55	152	813	0.00	46	137	813	0.00	52	128
$D20 \times 03 \times 11 - 9$	798	800	0.25	53	154	800	0.25	49	146	808	1.25	51	156
$D20 \times 03 \times 11 - 10$	1177	1210	2.80	48	123	1194	1.44	47	151	1204	2.29	57	133

Table 8: Detailed results for GRASP with $\alpha = 50\%$, $\alpha = 75\%$, and $\alpha = 100\%$ on instances D20 $\times$ 03 $\times$ 11.

		GRASP_NEH$_{sep}$ $\alpha = 50\%$				GRASP_NEH$_{sep}$ $\alpha = 75\%$				GRASP_NEH$_{sep}$ $\alpha = 100\%$ (Random Multistart)			
Instances	LB	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it	UB	g(%)	t(s)	it
$D20 \times 05 \times 11 - 1$	1071	1083	1.12	54	126	1080	0.84	47	150	1078	0.65	53	173
$D20 \times 05 \times 11 - 2$	1246	1259	1.04	53	160	1250	0.32	50	143	1255	0.72	51	162
$D20 \times 05 \times 11 - 3$	1226	1255	2.36	48	137	1250	1.96	54	172	1244	1.47	54	153
$D20 \times 05 \times 11 - 4$	1293	1296	0.23	53	134	1294	0.08	47	136	1293	0.00	51	168
$D20 \times 05 \times 11 - 5$	1214	1222	0.66	52	142	1216	0.16	47	144	1214	0.00	52	175
$D20 \times 05 \times 11 - 6$	1304	1330	1.99	54	135	1308	0.31	47	134	1308	0.31	51	166
$D20 \times 05 \times 11 - 7$	1388	1392	0.29	48	153	1389	0.07	47	129	1388	0.00	56	174
$D20 \times 05 \times 11 - 8$	1639	1674	2.13	52	149	1644	0.30	46	137	1662	1.40	53	164
$D20 \times 05 \times 11 - 9$	1215	1240	2.06	47	155	1240	2.06	49	156	1238	1.89	50	159
$D20 \times 05 \times 11 - 10$	1562	1568	0.38	54	162	1563	0.06	47	163	1562	0.00	52	164

Table 9: Detailed results for GRASP with $\alpha = 50\%$, $\alpha = 75\%$, and $\alpha = 100\%$ on instances D20 $\times$ 05 $\times$ 11.

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