Maintenance activities planning and grouping for complex structure systems

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Abstract: This paper presents a dynamic grouping maintenance strategy for complex systems whose structure may lead to both positive and negative economic dependence which imply that combining maintenance activities is cheaper (or more expensive respectively) than performing maintenance on components separately. Binary Particle Swarm Optimization (BPSO) algorithm is used to find optimal grouping planning which is NP-hard combinatorial problem. The proposed grouping maintenance strategy based on the rolling horizon approach can help to update the maintenance planning by taking into account short-term information which could be changed with time. A numerical example of a 10 components system is finally introduced to illustrate the use and the advantages of the proposed approach in the maintenance optimization framework.

Keywords: Grouping maintenance, Economic dependence, Multi-component system, Optimization.

1. INTRODUCTION

For multi-component systems, economic dependence which can be either positive or negative have been considered in maintenance optimization. Positive economic dependence implies that costs can be saved when several components are jointly maintained instead of separately. Negative economic dependence between components occurs when maintaining components simultaneously is more expensive than maintaining components individually. A number of maintenance optimization methods and maintenance strategies have been proposed and developed in the literature, see for example [1, 2, 3, 4]. In such papers, they focus mainly on series structures which lead directly to positive economic dependence [5, 6, 7, 8]. Recently, in [9, 10], both positive and negative economic dependence have been studied by introducing an opportunistic maintenance which attempts to balance the negative aspects and the positive aspects of grouping maintenance. The proposed maintenance models are however only applicable on limited class of systems with particular structure. From a practical point of view, with growing up quickly of industry, systems become more and more complex. Hence, the proposed models may no longer be used. The grouping maintenance problem remains widely open.

Moreover, in the grouping maintenance framework, static models with an infinite planning horizon are usually used in case of stable situations [5, 6, 7]. Recently, dynamic models have been introduced in order to change the planning rules according to short-term information (e.g. failures and varying deterioration of components) by using a rolling horizon approach [8]. The primary objective of this paper is thus to develop the rolling horizon approach for grouping maintenance of complex structure systems with both positive and negative dependence. For such systems, to find optimal grouping planning, dynamic programming (see [8, 3]) is no longer usable since the combinatorial problem can be formulated as a set partitioning problem, which however can be NP-hard due to the complex structure impact. To this end, the BPSO [11, 12, 13], recognized as a general search strategy which is simple for implementation and very efficient for global search, is used.

This paper is organized as follows. Section 2 is devoted to the description of general assumptions and grouping maintenance problem. The impact of system structure on grouping maintenance is also analyzed. Section 3 focuses on the development of the rolling horizon approach in the context of complex structure systems. A simple numerical example is introduced in Section 4. Finally, the last section presents the conclusions drawn from this work.

2. PROBLEM FORMULATION

2.1. General assumptions

Consider a complex structure coherent system consisting of N independent components in which component state is either operational or failure one. For the rate of occurrence of failures, denoted $r_i(t)$, we take a polynomial function with scale parameter $\lambda_i > 0$, and shape parameter $\beta_i > 1$:

$$r_i(t) = \frac{\beta_i}{\lambda_i}(\frac{t}{\lambda_i})^{\beta_i - 1} \tag{1}$$

Both corrective and preventive maintenance are considered for each component and we assume also that preventive and corrective maintenance durations can be neglected. According to the system structure, we consider two kinds of components: non-critical component (the system can be still functioning when the component stops); critical component (a shutdown of the component for whatever raison leads to a shutdown of the system). By definition, maintenance of a critical component may lead to an additional cost that could be relying on, for example, production quality loss or/and restart system costs... see [14]. Two kinds of shutdown costs are here considered:(i) planned shutdown cost, denoted C^p_{sys}, that is incurred when executing preventive maintenance on a critical component;(ii) unplanned shutdown cost, denoted C^u_{sys}, that is incurred when executing corrective maintenance on a critical component. Since the failure is random and preventive maintenance is planned, an unplanned shutdown is therefore more costly than a planned shutdown, $C^c_{sys} \geq C^p_{sys}$.

For component i ($i = 1, ..., N$), we assume that after a preventive maintenance (PM) action the component is considered "as good as new". The preventive maintenance cost can be divided into three parts: a fixed setup cost, denoted S, represents the common cost for all preventive maintenance activities. The setup cost depends only on the system. For example, the setup cost can be composed the cost of crew traveling and the preparation cost (erecting a scaffolding or opening a machine); a specific cost, denoted c^p_i, depending on the component; an additional cost C^p_{sys} is incurred if component i is critical. That mean that this additional cost depends on the component position in the system structure and the system characteristics. Let π_i is an indicator function ($\pi_i = 1$ if component i is a critical one, and otherwise $\pi_i = 0$ if component i is a not-critical one). As consequence, if a PM of component i is carried out, we have to pay the following cost:

$$C^p_i = S + c^p_i + \pi_i \cdot C^p_{sys} \tag{2}$$

To define the preventive maintenance cycle for each component, a long-term average cost based approach [15] is used and presented in Section 3. Between two preventive maintenance intervals, if component fails, a minimal repair is then immediately performed to restore the component to an operational state "as bad as old". Let c^r_i denote the specific repair cost of component i. As in the PM case, when a corrective action is carried out for component i, it requires a corrective maintenance cost, denoted C^c_i.

$$C^c_i = S + c^r_i + \pi_i \cdot C^u_{sys} \tag{3}$$

2.2. Grouping maintenance and the impact of the system structure

In the spirit of grouping maintenance strategies see [8, 1], grouping maintenance can reduce the maintenance cost. The setup cost can be shared when several maintenance activities are performed simultaneously since simultaneous maintenance executions on several (group) components require usually only one set-up cost [15, 3]. Note well, however that when a group of components are preventively performed together, the maintenance cost could be indirectly penalized with the reduction of components useful life if the maintenance dates are advanced and with the increasing of components failure probability which could imply a system immobilization if the maintenance dates are too late.

Moreover, the maintenance cost of a group may depend on the position of the group's components. We consider two kinds of groups: non-critical group (the system can be still functioning when all components of the group stop); critical group (the maintenance of the group leads to system stop). As consequence, maintenance on a critical group leads to a planned shutdown cost. Three following cases are considered: (i) if the critical group contains at least two critical components, the planned shutdown cost can then be shared; (ii) if the critical group contains one critical component, the planned shutdown cost remains then unchanged; (iii) if the critical group does not contain any critical component, the planned shutdown cost can then be penalized.

In order to find the optimal groups which could balance to minimize the system maintenance costs on the scheduling horizon, the rolling horizon approach introduced recently in [8, 15], will be developed by taking into account the system structure, see Section 3. An optimization algorithm, BPSO [11, 12], will be also used.

3. EXTENSION OF ROLLING HORIZON APPROACH

The developed rolling horizon approach consists of 5 phases: decomposition, tentative planning, economic profit formulations, grouping optimization using BPSO and rolling horizon.

3.1. Phase 1: Decomposition

For each individual component, we formulate an infinite horizon maintenance model to obtain the optimal individual preventive maintenance cycle that minimizes the component maintenance costs.

Let $M_i(x)$ denote the expected deterioration cost incurred in x time units since the latest operation on component i. According to the minimal repair policy, $M_i(x)$ can be expressed as the following:

$$M_i(x) = C_i^r \cdot \int_0^x r_i(y)dy. \tag{4}$$

From equations (1) and (4), we obtain: $M_i(x) = C_i^r \cdot (\frac{x}{\lambda_i})^{\beta_i}$

If component i is preventively maintained at x, the expected cost within interval $[0, x]$ is:

$$E_i(x) = C_i^p + M_i(x) = C_i^p + C_i^r \cdot (\frac{x}{\lambda_i})^{\beta_i} \tag{5}$$

By applying the renewal theory when executing the preventive maintenance of component i every x time units amount, the long-term average cost of component i can be determined as the following:

$$\phi_i(x) = \frac{E_i(x)}{x} = \frac{C_i^p + C_i^r \cdot (\frac{x}{\lambda_i})^{\beta_i}}{x} \tag{6}$$

Since $\phi_i(x)$ is a convex function, the optimal interval length for the preventive maintenances on component i, denoted x_i^*, can be obtained by setting the derivative of $\phi_i(x)$ to zero:

$$x_i^* = \arg\min_x \phi_i(x) = \lambda_i \sqrt[\beta_i]{\frac{C_i^p}{C_i^r(\beta_i - 1)}}. \tag{7}$$

And the minimal long-term average maintenance cost of component i:

$$\phi_i^* = \phi_i(x_i^*) = \frac{C_i^p \beta_i}{x_i^*(\beta_i - 1)} \tag{8}$$

The minimal long-term average maintenance cost of the system assuming that all components are optimally and individually maintained:

$$\phi_{sys}^* = \sum_{i=1}^{N} \phi_i^*. \tag{9}$$

The optimal interval length x_i^* which represents a nominal PM frequency of component i will be used to define tentative execution times in the tentative planning phase.

3.2. Phase 2: Tentative planning

The idea of this phase is to establish a tentative maintenance planning based on the individual preventive maintenance cycles.

Let t_{begin} denote the current date and i^j be jth maintenance of component i since t_{begin} and t_i^e is the operational time elapsed from the last preventive maintenance of component i before t_{begin}. Without loss generality we can set $t_{begin} = 0$. Based on the individual maintenance rules of phase 1, the current state of component i, the tentative execution time of operation (or activity) i^j, denoted t_{i^j}, is determined as follows, see [3]:

$$t_{i^1} = t_{begin} - t_i^e + x_i^*, \;\; t_{i^j} = t^*_{i^{j-1}} + x_i^* \text{ if } j > 1 \tag{10}$$

In order to evaluate the performance of grouping maintenance strategy, a finite planning horizon is usually defined according to the current date t_{begin} and the ending date t_{end} which guarantees that all components are preventively maintained at least one time in the horizon interval. It is shown in [3], t_{end} can be determined as follows: $t_{end} = \max_i t_{i^1}$.

After this phase, the tentative execution times of all individual maintenance activities in the scheduling horizon are identified. In the next phases we try to perform simultaneously several maintenance activities by changing their maintenance execution times to minimize total maintenance cost.

3.3. Phase 3: Economic profit formulations

The idea of this phase is to formulate the economic profits when preventive maintenance activities are simultaneously carried out. To this end, we will firstly find all cost-effective groups which can help to identify secondly an optimal grouping maintenance planning.

Cost-effective group Assume now a group of several different maintenance operations $i^j (i, j = 1, 2, ...)$, denoted G^k, are simultaneously performed at time t. The corresponding economic profit of this group can be divided into three parts as follows:

Since the execution of a group of m maintenance operations requires only one set-up cost, the group G^k yields a cost reduction:

$$U_{G^k} = (m-1) \cdot S \tag{11}$$

The penalty costs due to the changing of components execution maintenance date:

$$\triangle H^1_{G^k}(t) = \sum_{i^j \in G^k} h_i(t - t_{i^j}) = \sum_{i^j \in G^k} h_i(\triangle t_{i^j}), \tag{12}$$

$h_i(\triangle t_{i^j})$ is the penalty cost due to the movement execution date of the maintenance operation i^j. Since operation i^j is actually executed at time $t = t_{i^j} + \triangle t_{i^j}$ ($\triangle t_{i^j} > -x_i^*$) instead of t_{i^j}.

$$h_i(\triangle t_{i^j}) = E_i(x_i^* + \triangle t_{i^j}) - \big(E_i(x_i^*) + \triangle t_{i^j}.\phi_i^*\big) = C_i^r \cdot \Big(\frac{x_i^* + \triangle t_{i^j}}{\lambda_i}\Big)^{\beta_i} - C_i^r \cdot \Big(\frac{x_i^*}{\lambda_i}\Big)^{\beta_i} - \triangle t_{i^j} \cdot \frac{C_i^p \beta_i}{x_i^*(\beta_i - 1)} \tag{13}$$

The optimal execution time of the group G^k, denoted t_{G^k}, can be found when the $\triangle H^1_{G^k}(.)$ searches its minimal value $\triangle H^{1*}_{G^k}$. That is:

$$\triangle H^{1*}_{G^k} = \triangle H^1_{G^k}(t_{G^k}) = \min_t \sum_{i^j \in G^k} h_i(t_{G^k} - t_{i^j}) \tag{14}$$

If G^k is a critical one, the grouping can conduct to an additional cost that is expressed as follows:

$$\triangle H^2_{G^k} = C_{G^k} - C_{notG^k}, \tag{15}$$

The total planned shutdown costs when all components of the group are separately maintained:

$$C_{notG^k} = C^p_{sys} \cdot \sum_{i^j \in G^k} \pi_i; \tag{16}$$

Let π_{G^k} is an indicator function ($\pi_{G^k} = 1$ if G^k is a critique group, and $\pi_{G^k} = 0$ if G^k is a not-critique group). The planned shutdown cost when all components of the group are performed together:

$$C_{G^k} = C_{sys}^p \cdot \pi_{G^k} \tag{17}$$

Thanks to equations (15), (16) and (17), $\triangle H_{G^k}^2$ can be written as:

$$\triangle H_{G^k}^2 = C_{sys}^p \cdot (\pi_{G^k} - \sum_{i^j \in G^k} \pi_i). \tag{18}$$

Note well that if the group G^k is not critical, all components of the group are then not critical, $\triangle H_{G^k}^2$ is hence equal to zero. If G^k is a critical group, $\triangle H_{G^k}^2$ can be positive or negative depending on the number of critical components of the group (see again Section 2).

From Equations (11), (14), and (18), the economic profit of group G^k, denoted $EP(G^k)$, can be written as the following:

$$EP(G^k) = U_{G^k} - \triangle H_{G^k}^{1*} - \triangle H_{G^k}^2 \tag{19}$$

(i) If $EP(G^k) < 0$ the grouping maintenance is more expensive than performing maintenance on components separately, i.e. the grouping implies a negative economic dependence. As consequence, the components of the group should not be simultaneously maintained;

(ii) If $EP(G^k) \geq 0$, the maintenance cost can be saved when components are jointly maintained, i.e. grouping leads to a positive economic dependence. And the group is called cost-effective.

Grouping structure Based on all cost-effective groups, a grouping structure (or partition of all maintenance operations in the scheduling interval), denoted SGM, can be identified. As consequence, the total economic profit is calculated as follows:

$$EPS(SGM) = \sum_{G^k \in SGM} EP(G^k) = \sum_{G^k \in SGM} (U_{G^k} - \triangle H_{G^k}^{1*} - \triangle H_{G^k}^2). \tag{20}$$

An optimal grouping structure when the total economic profit searches its maximal value:

$$SGM^{optimal} = \arg\max_{SGM} EPS(SGM) \tag{21}$$

Note well that the maximum number of groups to be considered is here $2^n - 1$ for n maintenance activities. The finding of optimal grouping structure becomes difficult since the combinatorial problem can be formulated as a set partitioning problem, which however can be NP-hard. The dynamic programming algorithm proposed in [8, 3] can not therefore be used. To solve this problem, BPSO algorithm will be used and described in next paragraph.

3.4. Phase 4: Grouping optimization using BPSO

Binary particle swarm optimization was proposed by Kennedy and Eberhart in 1997, see [13], to solve optimization problems in binary search spaces (0 or 1). This method is loosely modeled on the flocking behavior of birds. A solution to a specific problem is presented by a position of a particle. A swarm of fixed number of particles is generated and each particle is initialized with a random position in the search space. Each particle flies through the search space with a velocity. At each iteration, a new velocity value of each particle is calculated based on its current velocity, the distance from its personal best (the best position of particle so far), and the distance from the global best (the best position of all particles). Once the velocity of each particle is updated, the particles are then moved to the new positions. The general structure of BPSO is presented in Fig 1.

Coding A coding phase determines how the problem is structured in the algorithm. In our problem, each grouping structure (a solution) is presented by a binary matrix of $(n-1)$ rows and n columns. Each row represents a group in which entry $(k, j) = 1$ if maintenance activity j is performed in group k and entry $(k, j) = 0$ for otherwise. For example considering a feasible solution of 5 preventive

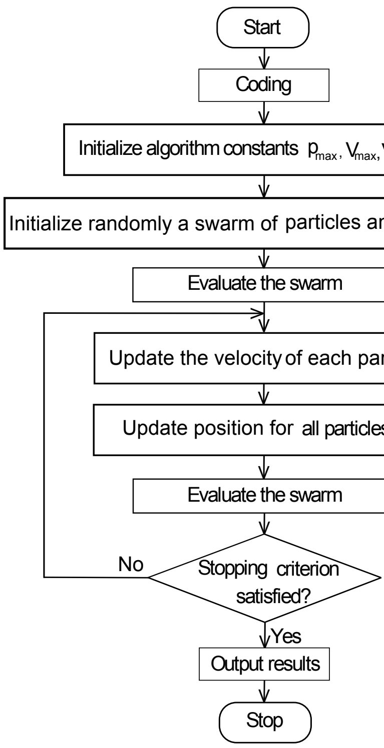

Figure 1: The general structure of BPSO

maintenance activities contains of 3 groups $G^1 = \{1, 2\}$, $G^2 = \{3, 4\}$, $G^3 = \{5\}$. This solution can be presented by the matrix (22a).

$$X = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} (22a) \qquad V_l^0 = \begin{pmatrix} -3.4 & -1.8 & 2.8 & 3.5 & 0.5 \\ -2.3 & -1.4 & -3.6 & 2.6 & 2.6 \\ 1.0 & 3.6 & -2.5 & 0.5 & 3.8 \\ -0.3 & 0.7 & -2.7 & 2.9 & 0.0 \end{pmatrix} (22b) \tag{22}$$

Create randomly a swarm of particles and its velocities A swarm of s particles is constructed for the BPSO. A small number of particles in the swarm may result in local convergence, on the contrary, a large number of particles will increase computational efforts and may make slow convergence. Therefore, the s is usually chosen between 60 and 100. The initial position of the lth particle is identified by a binary matrix, denoted X_l^0, whose elements are randomly assigned by 1 or 0.

To adjust the position of particles, the initial velocities of all particles are also randomly generated $(V_1^0, \ldots, V_s^0)$. Velocity values are restricted to the limit values (V_{max}, V_{min}) to prevent the particle from moving too rapidly from one region in search space to another. We usually set $V_{max} = 4$, $V_{min} = -4$. The initial velocity of the particle l, denoted V_l^0, is represented as matrix (22b).

Evaluate the swarm The goal of this step is to determine the personal best and global best in the swarm. To do that, we must firstly evaluate the position of each particle l at iteration pth according to its fitness function which is defined as follows:

$$f(X_l^p) = \begin{cases} -\infty & \text{if } \exists EP(G^k) < 0,\, G^k \in X_l^p \\ EPS(X_l^p) & \text{otherwise} \end{cases}$$

The personal best, denoted $ibest_l^p$, is the best position associated with the best fitness value of the particle l obtained until iteration p. $ibest_l^p$ is defined as follows:

$$ibest_l^p = \arg\max_q [f(X_l^q)], \text{where } q = 1 \div p$$

The global best at iteration p, denoted $gbest^p$, is the best position among all particles in the swarm, which is achieved so far and can be expressed as follows:

$$gbest^p = \arg\max_l [f(ibest_l^p)], \text{where } l = 1 \div s$$

Update the velocity of each particle The objective of this step is to calculate the new value of velocity of each particle to determine the next position of the particle. Based on the current velocity value, the personal best and the global best, the velocity of each particle can be updated as follows:

$$V_l^{p+1}(k,j) = V_l^p(k,j) + r_1 \cdot U_1 \cdot [ibest_l^p(k,j) - X_l^p(k,j)] + r_2 \cdot U_2 \cdot [gbest^p(k,j) - X_l^p(k,j)]$$

U_1, U_2 are uniform random numbers between $(0,1)$; r_1, r_2 are social and cognitive parameters which are taken as $r_1 = r_2 = 2$ consistent with the literature, see [12].

In BPSO, the velocities of the particles are defined in terms of probabilities that a element of matrix X_l^p will change to one. Using this definition a velocity must be restricted within the range $[0,1]$. So whenever a velocity value is computed, the following piece-wise function, whose range is closed interval $[V_{min}, V_{max}]$, is used to restrict them to minimum and maximum value.

$$h(V_l^{p+1}(k,j)) = \begin{cases} V_{max} & \text{if } V_l^{p+1}(k,j) > V_{max} \\ V_l^{p+1}(k,j) & \text{if } |V_l^{p+1}(k,j)| < V_{max} \\ V_{min} & \text{if } V_l^{p+1}(k,j) < V_{min} \end{cases}$$

After applying the piece-wise linear function, the following sigmoid function is used to scale the velocities between 0 and 1, which is then used for converting them to the binary values.

$$sigmoid(V_l^{p+1}(k,j)) = \frac{1}{1 + e^{-V_l^{p+1}(k,j)}}$$

Update position for all particles Based on new velocities obtained and let U is uniform random numbers between $(0,1)$, we update the new position for each particle by using the equation below:

$$X_l^{p+1}(k,j) = \begin{cases} 1 & \text{if } U < sigmoid(V_l^{p+1}(k,j)) \\ 0 & \text{otherwise} \end{cases}$$

Stopping criterion Stopping criterion is introduced to stop the algorithm process. Herein, limited iterations number (p_{max}) is used as a stopping criteria.

3.5. Phase 5: Rolling horizon

The maintenance manager can change the planning when some new information appears or when a planning for a new period and go back to step 4. Due to the previous phase, we have an optimal grouping structure within the finite planning horizon $[t_{begin}, t_{end}]$. However, with time some new information (like maintenance resources constraints relying on for example management and new technology, opportunities, ...) by which this optimal grouping structure can be impacted may become available. To update the maintenance planning, a new optimal grouping structure within a new period must be identified. To this end, we simply go back to phase 2.

4. NUMERICAL EXAMPLES

The purpose of this section is to show how the proposed grouping maintenance strategy can be used in preventive maintenance optimization of complex systems. The impact of the system structure on the grouping maintenance planning will be analyzed. A short comparison with previous grouping approach [8] will be also discussed.

Consider a 10 components system whose structure is shown in Figure 2. When a component fails, it is immediately maintained according to a minimal-repair policy. Corrective maintenance restores the component involved into a state "as bad as old". We assume failure rate of a component i ($i = 1, \dots, 10$) is described by a Weibull distribution with scale parameter $\lambda_i > 0$, and shape parameter $\beta_i > 1$. Table 1 reports the random data for 10 components. For set-up cost and shutdown system costs, we take $S = 10$ and $C_{sys}^p = 40$, $C_{sys}^u = 45$ respectively.

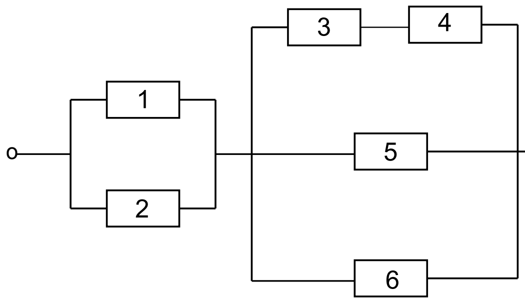

Figure 2: Structure system

Table 1: Data of 10 components

Component i	1	2	3	4	5	6	7	8	9	10
λ_i	259	270	280	249	297	260	285	285	250	260
β_i	1.90	2.00	2.00	1.95	1.95	2.00	2.00	2.00	1.90	2.00
c_i^p	115	125	125	105	145	125	165	145	125	135
c_i^r	42	32	22	35	40	35	20	20	40	35
t_i^e	184.37	214.07	295.11	153.61	364.70	150.33	472.54	459.55	155.71	226.71

4.1. Maintenance planning with negligible system structure

Consider now maintenance strategies in which the system structure is not considered. We assume that the system is stopped when performing maintenance on any component, i.e. $\pi_i = 1, \forall i = (1, \ldots, 10)$.

Individual maintenance planning We consider an optimal maintenance planning in which all components' preventive maintenance are separately performed. To define the optimal preventive maintenance cycle of components, the long-term maintenance cost model was used. C_i^p, C_i^c, x_i^*, ϕ_i^* and t_i^1 are calculated by substitution of the data input and π_i in Equations (2),(3),(7),(6),(10) respectively. All results are shown in table 2.

Table 2: Values of C_i^p, C_i^c, x_i^*, ϕ_i^* and t_i^1 with negligible system structure

Component i	1	2	3	4	5	6	7	8	9	10
π_i	1	1	1	1	1	1	1	1	1	1
C_i^p	165	175	175	155	195	175	215	195	175	185
C_i^r	97	87	77	90	95	90	75	75	95	90
x_i^*	362.09	382.93	422.12	337.83	440.90	362.55	482.54	459.55	364.47	372.77
ϕ_i^*	0.9620	0.9140	0.8292	0.9418	0.9078	0.9654	0.8911	0.8487	1.0136	0.9926
t_i^1	177.75	168.87	127.01	184.22	76.20	212.22	10.00	0.00	208.76	146.05

The average maintenance cost of the system when all components are individually maintained is finally:

$$\phi_{sys}^{\overline{SG}} = \sum_{i=1}^{N} \phi_i^* = 9.2662 \tag{23}$$

Grouping maintenance planning To establish an optimal grouping maintenance, the proposed grouping maintenance strategy was used. Based on the components' individual maintenance execution date, the scheduling horizon is [0, 212.22] ($t_{begin} = 0$, $t_{end} = \max t_i^1 = 212.22$). After 500 iterations, BPSO optimization algorithm provided an optimal grouping maintenance planning. The results are reported in Table 3. The total cost saving is $EPS = 393.9172$ which is equivalent to the average saving $\Delta\phi_{sys} = \frac{EPS}{t_{end}} = \frac{393.9172}{212.22} = 1.8562$. Hence, the average maintenance cost of the system with proposed grouping maintenance planning is:

$$\phi_{sys}^{\overline{S}G} = \phi_{sys}^{\overline{SG}} - \Delta\phi_{sys} = 7.41 \tag{24}$$

The result shows that grouping maintenance is cheaper than performing maintenance on components separately. Note well that under the assumptions above, the system can be considered as a series structure for which the rolling horizon approach [8] can be used. By applying this approach, we obtain the same results shown in Table 3.

Table 3: Optimal grouping planning with negligible system structure

Group	Group components	Group execution date (t_G)	Group cost saving
1	1,2,3,4,5,6,7,8,9,10	140.47	393.9172

4.2. Maintenance planning with taking into account the system structure

Individual maintenance planning According to the system structure, indicator functions π_i $(i = 1, ..., 10)$ can be easily determined. In the same manner, C_p^i, C_i^r, x_i^*, ϕ_i^*, t_i^1 are then calculated and showed in Table 4.

The average maintenance cost of the system when all components are individually maintained is finally:

$$\phi_{sys}^{S\overline{G}} = \sum_{i=1}^{N} \phi_i^* = 6.3897 \tag{25}$$

The result shows that the consideration of the system structure in maintenance can save the maintenance cost. If the system structure is taken into account, the individual maintenance planning is even cheaper than an optimal grouping maintenance planning in which the system structure is ignored.

Table 4: Values of individual optimization with taking into account the system structure

Component i	1	2	3	4	5	6	7	8	9	10
π_i	0	0	0	0	0	0	1	1	0	0
C_i^p	125	135	135	115	155	135	215	195	135	145
C_i^r	52	42	32	45	50	45	75	75	50	45
x_i^*	434.37	484.07	575.11	413.61	544.70	450.33	482.54	459.55	445.71	466.71
ϕ_i^*	0.6075	0.5578	0.4695	0.5707	0.5841	0.5996	0.8911	0.8487	0.6394	0.6214
t_i^1	250	270	280	260	180	300	10	0	290	240

Grouping maintenance planning In order to establish a grouping maintenance planning, we used again the proposed grouping approach. According to the components' individual maintenance execution date, the planning horizon is [0, 300]. Table 5 reports the results obtained by using BPSO optimization algorithm with 500 iterations.

Table 5: Optimal grouping planning with taking into account the system structure

Group	Group components	Group execution date (t_G)	Group cost saving
1	1,5,10	226.60	18.4307
2	2,3,4,6,9	280.31	39.3498
3	7,8	5.00	49.9538

The total maintenance savings cost is $EPS = 18.4307 + 39.3498 + 49.9538 = 107.7343$ and the maintenance cost saving par time unit is:

$$\phi_{sys}^{SG} = \phi_{sys}^{S\overline{G}} - \frac{107.7343}{300} = 6.0306. \tag{26}$$

From (23), (24), (25) and (26), we obtain an important ranking for the average maintenance cost of the system:

$$\phi_{sys}^{SG} > \phi_{sys}^{S\overline{G}} > \phi_{sys}^{\overline{S}G} > \phi_{sys}^{\overline{SG}}.$$

The results assert that grouping can save the maintenance cost and that the system structure can take an important rule and should be taken into account in maintenance planning optimization.

5. CONCLUSIONS

In this work, the rolling horizon approach introduced recently in [8] is developed for grouping maintenance planning of complex systems whose structure can take both positive and negative impact on maintenance grouping. The numerical results show that taking into account the system structure into grouping maintenance procedure may reduce significantly maintenance cost. In order to find an optimal grouping planning, Binary Particle Swarm Optimization algorithm is proposed to use. This optimization algorithm can help to solve combinatorial grouping problem which can be NP-hard.

Our future research work will focus on the development of the proposed method for systems which have the stochastically and structurally dependent between components and also with the taking into account the duration of maintenance activities.

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